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# STRINGS AS A MODEL FOR PARENT AND BABY UNIVERSES: TOTAL SPLITTING RATES

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## Abstract

Emission of hard microscopic string (graviton) by an excited macroscopic string may be viewed as a model of branching of a  $(1+1)$ -dimensional baby universe off large parent one. We show that, apart from a trivial factor, the total emission rate is not suppressed by the size of the macroscopic string. This implies unsuppressed loss of quantum coherence in  $(1+1)$ -dimensional parent universe.

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**1.** Theory of fundamental strings in critical dimension may serve as a model for parent and baby universes [1, 2, 3]. Long smooth strings carrying particle-like excitations may be viewed as large  $(1+1)d$  universes, while microscopic string states (gravitons and alike) model baby universes. Within this model, one may try to understand various issues which were originally discussed in the context of  $(3+1)$ -dimensional theory of gravity [4, 5, 6, 7, 8, 9, 10] (for further motivation see ref. [3], which we will refer to as I in this paper). In particular, it has been argued in I that the emission of a baby universe (hard graviton) into  $D$ -dimensional target space-time, induced by “particles” in the large universe (macroscopic string), leads to the loss of quantum coherence for  $(1+1)d$  observer living in the parent universe. It is then of interest to estimate the dependence of the corresponding emission rate on the size of the parent universe,  $L$ . From  $D$ -dimensional point of view this is the rate of the decay of a slightly excited macroscopic string into a graviton and another macroscopic string. The results of I concerning this rate were not decisive: only special final states of the macroscopic string were considered, and the partial decay rates into these particular final states were suppressed at large  $L$ , albeit only logarithmically.

In this paper we show that the *total* rate of the emission of hard gravitons by excited macroscopic string is unsuppressed at large  $L$  (apart from a trivial factor). In  $(1+1)d$  language this means that the branching off of baby universes, induced by interactions of  $(1+1)d$  “particles”, is finite for parent universes of large size. As argued in I, this in turn implies that the loss of quantum coherence in the large  $(1+1)d$  universe occurs at finite,  $L$ -independent rate.

**2.** The construction of excited macroscopic states of bosonic closed string in critical dimension, outlined in I, is as follows. One considers  $D$ -dimensional target space with one dimension,  $X^1$ , compactified to a large circle of length  $2\pi L$ . Let  $|\mathcal{P}\rangle$  be the ground state of the string winding once around this compact dimension. In its rest frame

$$\mathcal{P} = (M_0, \mathbf{0}) \tag{1}$$

with

$$M_0^2 = 4L^2 - 8$$

(we follow conventions of ref. [11]). The excited string states are constructed by making use of the DDF operators

$$a_n^\alpha = \int_0^\pi \frac{d\sigma_+}{\pi} \exp \left[ 4in \frac{e_\mu X_L^\mu(\sigma_+)}{e_\mu P_L^\mu} \right] \xi_i^\alpha \partial_+ X_L^i(\sigma_+),$$

$$\tilde{a}_{\tilde{n}}^\alpha = \int_0^\pi \frac{d\sigma_-}{\pi} \exp \left[ 4i\tilde{n} \frac{e_\mu X_R^\mu(\sigma_-)}{e_\mu P_R^\mu} \right] \xi_i^\alpha \partial_- X_R^i(\sigma_-),$$

where  $e^\mu$  is an arbitrary (but fixed) light-like vector in  $D$  dimensions with  $e^0 = 1$ ,  $\xi^\alpha$  ( $\alpha = 1, \dots, D-2$ ) are orthonormal spatial vectors which are orthogonal to  $e$ ,

$$P_L^\mu = P^\mu + 2L^\mu, \quad P_R^\mu = P^\mu - 2L^\mu$$

with  $L^\mu = (0, L, 0, \dots, 0)$ , and  $X_{L,R}^\mu$  are usual left and right components of the string coordinate operator in the sector of winding string [11], e.g.,

$$X_L^\mu(\sigma_+) = \frac{1}{2}X^\mu + \frac{1}{2}P_L^\mu\sigma_+ + \frac{i}{2} \sum_{k \neq 0} \frac{1}{k} \alpha_k^\mu e^{-2ik\sigma_+}.$$

The macroscopic string state with two particle-like excitations is

$$|n, \alpha; \tilde{n}, \beta\rangle = \frac{1}{\sqrt{n\tilde{n}}} a_{-n}^\alpha \tilde{a}_{-\tilde{n}}^\beta |\mathcal{P}\rangle, \quad (2)$$

where normalization corresponds to “one particle per volume  $L$ ” in  $(1+1)$  dimensions. The state (2) obeys the Virasoro constraints provided that

$$\frac{n}{(e\mathcal{P}_L)} = \frac{\tilde{n}}{(e\mathcal{P}_R)} = \frac{n + \tilde{n}}{2(e\mathcal{P})}.$$

Its target space momentum and mass are

$$P^\mu = \mathcal{P}^\mu - \frac{2(n + \tilde{n})}{(e\mathcal{P})} e^\mu, \quad (3)$$

$$M^2 = -P^2 = M_0^2 + 4(n + \tilde{n}).$$

Roughly speaking, the state (2) contains one particle moving right in  $(1+1)$ -dimensional universe and one particle moving left, the bare  $(1+1)d$  momenta of these particles being  $n/L$  and  $(-\tilde{n}/L)$ , respectively<sup>3</sup>. Therefore, we will be interested in the regime

$$L \rightarrow \infty, \quad \frac{n}{L}, \frac{\tilde{n}}{L} = \text{fixed}, \quad (4)$$

which corresponds to large universe with “particles” of finite  $(1+1)d$  bare momenta.

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<sup>3</sup>Of course, this interpretation should not be taken literally, as we are dealing with conformal field theory in  $(1+1)$  dimensions. Nevertheless, we will use somewhat loose notion of “particles” in what follows, since the analogy to usual particles is close enough.

**3.** The main purpose of this paper is to evaluate the  $L$ -dependence, in the regime (4), of the total decay rate of the state (2) into another excited macroscopic string state and one graviton whose target space momentum is finite at large  $L$  (in this sense the graviton is hard). This rate is proportional to the imaginary part of the forward amplitude shown in fig.1.

The amplitude can be written as follows

$$A = \frac{\kappa^2}{4\pi} \int d^D Q \frac{\Pi_{\mu\nu\rho\sigma}}{Q^2} \langle n, \alpha; \tilde{n}, \beta | V_{\mu\nu}(-Q) \Delta V_{\rho\sigma}(Q) | n, \alpha; \tilde{n}, \beta \rangle, \quad (5)$$

where  $\Pi_{\mu\nu\rho\sigma}(Q) = \eta_{\mu\nu}\eta_{\rho\sigma} + \dots$  is the polarization factor in the graviton propagator,

$$V_{\mu\nu}(Q) = \partial_+ X^\mu \partial_- X^\nu e^{-iQX}$$

is the vertex operator for graviton, and  $\Delta$  is the string propagator. Hereafter the standard  $i\epsilon$  prescription is assumed in denominators. The evaluation of the matrix element in eq.(5) is tedious but straightforward. One finds

$$\begin{aligned} A &= \frac{\kappa^2}{4\pi} \xi_i^\alpha \xi_{i'}^\alpha \xi_j^\beta \xi_{j'}^\beta \int d^D Q \frac{\Pi_{\mu\nu\rho\sigma}}{Q^2} \int dz d\bar{z} B_{L,\mu\rho}^{ii'}(z) B_{R,\nu\sigma}^{jj'}(\bar{z}) \\ &\times z^{-\frac{1}{4}(PQ) - \frac{1}{2}(QL) + \frac{1}{8}Q^2 - 1} (1-z)^{-\frac{1}{4}Q^2} \\ &\times (\bar{z})^{-\frac{1}{4}(PQ) + \frac{1}{2}(QL) + \frac{1}{8}Q^2 - 1} (1-\bar{z})^{-\frac{1}{4}Q^2}, \end{aligned} \quad (6)$$

where

$$\begin{aligned} B_{L,\mu\rho}^{ii'}(z) &= \frac{1}{n} \int \frac{du}{2\pi} \frac{du'}{2\pi} \frac{1}{u^{n+1}} \frac{1}{(u')^{n+1}} \\ &\times (1-u)^{-a} \left(1 - \frac{u}{z}\right)^a (1-u')^a (1-zu')^{-a} C_{\mu\rho}^{ii'}(z, u, u'). \end{aligned} \quad (7)$$

Here

$$a = \frac{n + \tilde{n}}{2(e\mathcal{P})}(eQ),$$

and integration contours in complex  $u$ - and  $u'$ -planes are small circles around the origin. The expression for  $C_{\mu\rho}^{ii'}$  is not particularly illuminating; it is given in Appendix.

Expression similar to eq.(7) may be written also for  $B_{R,\nu\sigma}^{jj'}(\bar{z})$ .

The imaginary part of the amplitude emerges in the following way. As is clear from eqs.(7) and (21),  $B_L(z)$  entering eq.(6) can be represented as finite sum (we omit

indices  $i, i', \mu, \rho$  in what follows)

$$B_L(z) = \sum_{m=-n}^n B_{L,m} z^{-m}$$

with real coefficients  $B_{L,m}$ . Similarly,

$$B_R(\bar{z}) = \sum_{\tilde{m}=-\tilde{n}}^{\tilde{n}} B_{R,\tilde{m}}(\bar{z})^{-\tilde{m}}.$$

At given  $m$  and  $\tilde{m}$ , the integral over  $z, \bar{z}$  is straightforward to evaluate (one possibility is to make use of the relation between closed and open string amplitudes [11]). One finds

$$\begin{aligned} A &= \frac{\kappa^2}{4\pi} \xi \cdot \xi \cdot \xi \cdot \xi \cdot \xi \sum_{m,\tilde{m}} \int d^D Q \frac{\Pi}{Q^2} B_{L,m} B_{R,\tilde{m}} \sin\left(\frac{\pi Q^2}{4}\right) \\ &\times \frac{\Gamma\left(-\frac{1}{4}(PQ) - \frac{1}{2}(QL) - m + \frac{1}{8}Q^2\right) \Gamma\left(1 - \frac{1}{4}Q^2\right)}{\Gamma\left(-\frac{1}{4}(PQ) - \frac{1}{2}(QL) - m - \frac{1}{8}Q^2 + 1\right)} \\ &\times \frac{\Gamma\left(\frac{1}{4}(PQ) - \frac{1}{2}(QL) + \tilde{m} + \frac{1}{8}Q^2\right) \Gamma\left(1 - \frac{1}{4}Q^2\right)}{\Gamma\left(\frac{1}{4}(PQ) - \frac{1}{2}(QL) + \tilde{m} - \frac{1}{8}Q^2 + 1\right)}. \end{aligned} \quad (8)$$

The imaginary part of this integral is obviously due to the region  $Q^2 \sim 0$  (graviton mass shell). Since the explicit factor  $Q^2$  from the graviton propagator is cancelled by  $\sin\left(\frac{\pi Q^2}{4}\right)$ , the imaginary part appears when the product of gamma functions has double pole (Cutkosky rule). This occurs when

$$\begin{aligned} \frac{1}{4}(PQ) + \frac{1}{2}(QL) + m &= 0, \\ \frac{1}{4}(PQ) - \frac{1}{2}(QL) + \tilde{m} &= 0. \end{aligned} \quad (9)$$

Two remarks are in order. First, the compact component of the graviton momentum is quantized in units  $1/L$  [11], i.e.,

$$(QL) = r = \text{integer}.$$

Equation (9) then implies

$$\tilde{m} = m + r, \quad (10)$$

i.e.,  $\tilde{m}$  is fixed for given  $m$  and  $Q$ . Second, eq.(9) corresponds to the mass shell condition for the intermediate macroscopic string state, as it should be. Indeed,

eqs.(9) and (3) imply that the momentum of the intermediate state,  $P' = P - Q$ , obeys the following relations,

$$\begin{aligned} -\frac{1}{4}(P')^2 &= (n - m) + (\tilde{n} - \tilde{m}) + \frac{1}{4}M_0^2, \\ (P'L) &= (\tilde{n} - \tilde{m}) - (n - m), \end{aligned} \quad (11)$$

which are precisely the mass shell conditions [11] for physical state of a string winding around the compact dimension, with mode numbers  $(n - m)$  and  $(\tilde{n} - \tilde{m})$ .

In the region (9), the integrand in eq.(8) becomes

$$-\frac{\pi}{4}B_{L,m}B_{R,\tilde{m}} \frac{1}{\frac{1}{4}(PQ) + \frac{1}{2}(QL) + m - \frac{1}{8}Q^2} \frac{1}{\frac{1}{4}(PQ) - \frac{1}{2}(QL) + \tilde{m} + \frac{1}{8}Q^2}.$$

To obtain the imaginary part, we recall the Cutkosky rule and obtain

$$\text{Im}A = 4\pi^2\kappa^2 \sum_m \int d^DQ \Pi(Q)B_{L,m}B_{R,\tilde{m}}\delta((PQ) + 2(QL) + 4m)\delta(Q^2), \quad (12)$$

where  $\tilde{m}$  is given by eq.(10).

Let us now evaluate the dependence of  $\text{Im}A$  on the length  $L$  in the regime (4). We begin with  $B_{L,m}$ . It has the following representation (according to eq.(11), positive  $m$  are of interest),

$$B_{L,m} = \frac{1}{2\pi i} \int d\zeta \frac{1}{\zeta^{m+1}} B_L \left( z = \frac{1}{\zeta} \right),$$

where the integration contour runs around the origin in complex  $\zeta$  plane. From eq.(7) we see that

$$\begin{aligned} B_{L,m} &= \frac{1}{i} \frac{1}{n} \int \frac{d\zeta}{2\pi} \frac{du}{2\pi} \frac{du'}{2\pi} \frac{1}{\zeta^{m+1}} \frac{1}{u^{n+1}} \frac{1}{(u')^{n+1}} \\ &\times (1-u)^{-a} (1-u\zeta)^a (1-u')^a \left(1 - \frac{u'}{\zeta}\right)^{-a} C \left( z = \frac{1}{\zeta}, u, u' \right). \end{aligned} \quad (13)$$

Now, recall that

$$n \propto L$$

and also

$$m \propto L$$

(the latter relation, valid for  $Q$  independent of  $L$ , follows from eq.(9) and  $P \propto L$ ). Therefore, we may use the following asymptotic estimate

$$\int dz_1 \dots dz_k z_1^{-\lambda a_1} \dots z_k^{-\lambda a_k} \Pi(1 - z_p)^{\alpha_p} \Pi(1 - z_p z_q)^{\beta_{pq}} \Pi(1 - z_p/z_q)^{\gamma_{pq}} \\ \propto \left(\frac{1}{\lambda}\right)^{\sum \alpha_p + \sum \beta_{pq} + \sum \gamma_{pq} + k}, \quad (14)$$

which is valid as  $\lambda \rightarrow \infty$  with  $a_p, \alpha_p, \beta_{pq}, \gamma_{pq}$  fixed. This estimate is similar to ones used for evaluation of multi-Regge behavior of dual amplitudes, and can be obtained in a way parallel to that of ref. [12]. Applying this estimate to the integral in eq.(13) we see that for obtaining the  $L$ -dependence of  $B_{L,m}$  we have only to count factors like

$$\frac{1}{1-u}, \quad \frac{1}{1-u'}, \quad \frac{1}{1-uu'}, \quad \frac{1}{1-u/z}, \quad \frac{1}{1-zu'}, \quad (15)$$

as well as explicit  $L$ -dependent factors, in  $C(z, u, u')$ : each factor of the form (15) and each factor  $P$  produces a factor  $L$ , while factors like  $Q^\mu, z, u, u'$  do not matter for the asymptotic behavior of  $B_{L,m}$ . Making use of eq.(21) we obtain

$$B_{L,m} \propto \frac{1}{n} \left(\frac{1}{L}\right)^3 L^4,$$

where the first factor is explicit in eq.(13), the second factor is due to three integrations ( $k = 3$  in the notation of eq.(14)), and the last factor is obtained by counting factors (15) and explicit  $L$ -dependent factors in  $C(z, u, u')$ . So, we find that  $B_{L,m}$  is independent of  $L$  in the limit of large  $L$ .

The remaining factors of  $L$  in  $\text{Im}A$ , eq.(12), come from two sources. First, we have a factor  $L^{-1}$  from

$$\delta((PQ) + 2(QL) + 4m) \propto \frac{1}{L} \delta\left(Q^0 - Q^1 - \frac{2m}{L}\right)$$

(because  $P = (2L, 0, \dots, 0) + O(1)$ ). Second, summation over  $m$  gives a factor  $L$ ,

$$\sum_m = L \int d\left(\frac{m}{L}\right)$$

(in other words, density of states of the final macroscopic string is of order  $L$ ). Combining these two factors, we see that

$$\text{Im}A = \text{independent of } L.$$

Finally, the graph of fig.1 represents the correction to  $(mass)^2$  of the macroscopic string, and the decay rate  $\Gamma$  is related to  $\text{Im}A$  as follows,

$$\Gamma = \text{Im} (mass) = \frac{1}{2M} \text{Im} (mass)^2 \propto \frac{1}{L} \text{Im}A.$$

So, we find<sup>4</sup>

$$\Gamma \propto \frac{1}{L}. \quad (16)$$

This behavior of the decay rate is precisely what one expects from the point of view of  $(1+1)$  dimensions. Indeed, the state (5) is normalized to contain two “particles” in the  $(1+1)d$  universe of size  $L$ . The interaction rate of these particles is of order  $1/L$ , in agreement with eq.(16). We conclude that the emission of a baby universe induced by interactions of “particles” has no intrinsic suppression by the size of the parent universe.

**4.** String model of  $(1+1)$ -dimensional large and microscopic universes provides an example of a theory where branching off of baby universes induces the loss of quantum coherence in the parent universe. The results of this paper show that the rate of this loss of coherence is unsuppressed by the size of parent universes. It remains to be understood whether this property is a peculiarity of the string model, or it is generic to all theories allowing for wormholes/baby universes.

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### Appendix.

For completeness, we present here the amplitude shown in fig.2.

The initial and final states of the macroscopic string are DDF states with two “particles” in each of them; the momenta of the initial and final smooth strings,  $\mathcal{P}$  and  $\mathcal{P}'$  and the sets of vectors  $(e_\mu, \xi_\alpha)$  and  $(e'_\mu, \xi'_\alpha)$ , used for the construction of the DDF operators, need not be the same for initial and final states. The amplitude is written as follows,

$$A = \frac{\kappa^2}{4\pi} \zeta_{\mu\nu}(Q) \zeta_{\rho\sigma}(K)$$

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<sup>4</sup>Our estimate of  $B_{L,m}$  and  $B_{R,\bar{m}}$ , on which eq.(16) is based, would not be valid if there were cancellations between contributions of various terms in  $C(z, u, u')$ . In that case the rate would be of order  $L^{-2}$  or smaller. However, the results of I imply that the partial decay rate into *some* final states of macroscopic string is of order  $L^{-1}(\ln L)^{-1}$ , so the total decay rate is at least of this order. This excludes the dangerous cancellations and ensures that eq.(16) is indeed correct.



$$\begin{aligned}
& \times \left( \langle n', \alpha'; \tilde{n}', \beta' | V_{\mu\nu}(-Q) \Delta V_{\rho\sigma}(K) | n, \alpha; \tilde{n}, \beta \rangle \right. \\
& \left. + \langle n', \alpha'; \tilde{n}', \beta' | V_{\rho\sigma}(K) \Delta V_{\mu\nu}(-Q) | n, \alpha; \tilde{n}, \beta \rangle \right), \tag{17}
\end{aligned}$$

where  $\zeta(K)$  and  $\zeta(Q)$  are polarizations of the incoming and outgoing gravitons, respectively. The amplitude shown in fig.1 is obtained from eq.(17) by setting

$$K = Q, \quad n = n', \quad \tilde{n} = \tilde{n}', \quad \mathcal{P} = \mathcal{P}', \quad e = e', \quad \xi = \xi', \tag{18}$$

substituting  $\zeta \cdot \zeta$  by the graviton propagator and integrating over  $Q$ .

The evaluation of the amplitude (17) is straightforward. One finds

$$\begin{aligned}
A &= \frac{\kappa^2}{4\pi} \zeta_{\mu\nu}(Q) \zeta_{\rho\sigma}(K) \xi_i^\alpha \xi_{i'}^{\alpha'} \xi_j^\beta \xi_{j'}^{\beta'} \int dz d\bar{z} B_{L,\mu\rho}^{ii'}(z) B_{R,\nu\sigma}^{jj'}(\bar{z}) \\
&\times z^{-\frac{1}{4}(PK) - \frac{1}{2}(KL) + \frac{1}{8}K^2 - 1} (1 - z)^{-\frac{1}{4}(KQ)} \\
&\times (\bar{z})^{-\frac{1}{4}(PK) + \frac{1}{2}(KL) + \frac{1}{8}K^2 - 1} (1 - \bar{z})^{-\frac{1}{4}(KQ)}, \tag{19}
\end{aligned}$$

where

$$\begin{aligned}
B_{L,\mu\rho}^{ii'}(z) &= \frac{1}{\sqrt{n n'}} \int \frac{du}{2\pi} \frac{du'}{2\pi} \frac{1}{u^{n+1}} \frac{1}{(u')^{n'+1}} \\
&\times (1 - u)^{-a} \left(1 - \frac{u}{z}\right)^b (1 - u')^{a'} (1 - zu')^{-b'} \\
&\times (1 - uu')^{-c} C_{\mu\rho}^{ii'}(z, u, u') \tag{20}
\end{aligned}$$

with

$$\begin{aligned}
a &= \frac{n + \tilde{n}}{2(e\mathcal{P})}(eQ), & b &= \frac{n + \tilde{n}}{2(e\mathcal{P})}(eK), \\
a' &= \frac{n' + \tilde{n}'}{2(e'\mathcal{P}')}(e'Q), & b' &= \frac{n' + \tilde{n}'}{2(e'\mathcal{P}')}(e'K), \\
c &= \frac{n + \tilde{n}}{(e\mathcal{P})} \frac{n' + \tilde{n}'}{(e'\mathcal{P}')} (ee').
\end{aligned}$$

The factor  $C_{\mu\rho}^{ii'}$  has the following form

$$\begin{aligned}
C_{\mu\rho}^{ii'}(z, u, u') = & D_1^i D_2^{i'} R_1^\mu R_2^\rho + D_1^i R_2^\rho A_1^{i'\mu} + D_1^i R_1^\mu A_2^{i'\rho} + R_1^\mu R_2^\rho F^{ii'} \\
& + D_2^{i'} R_1^\mu B_1^{i\rho} + D_2^{i'} R_2^\rho B_2^{i\mu} + D_1^i D_2^{i'} G^{\mu\rho} \\
& + A_1^{i'\mu} B_1^{i\rho} + A_2^{i'\rho} B_2^{i\mu} + F^{ii'} G^{\mu\rho},
\end{aligned} \tag{21}$$

where

$$\begin{aligned}
D_1^i &= \frac{1}{2} \mathcal{P}_L^i - \frac{1}{2} Q^i \frac{u}{1-u} + \frac{1}{2} K^i \frac{u/z}{1-u/z} - \frac{2n'e^i}{(e'\mathcal{P}'_L)} \frac{uu'}{1-uu'}, \\
D_2^{i'} &= \frac{1}{2} \mathcal{P}'^{i'}_L + \frac{1}{2} Q^{i'} \frac{u'}{1-u'} - \frac{1}{2} K^{i'} \frac{u'z}{1-u'z} - \frac{2ne^{i'}}{(e\mathcal{P}_L)} \frac{uu'}{1-uu'}, \\
R_1^\mu &= \frac{1}{4} (P_L^\mu + P'^\mu_L) - \frac{n+\tilde{n}}{2(e\mathcal{P})} e^\mu \frac{1+u}{1-u} - \frac{n'+\tilde{n}'}{2(e'\mathcal{P}')} e'^\mu \frac{1+u'}{1-u'} - \frac{1}{2} K^\mu \frac{z}{1-z}, \\
R_2^\rho &= \frac{1}{4} (P_L^\rho + P'^\rho_L) - \frac{n+\tilde{n}}{2(e\mathcal{P})} e^\rho \frac{1+u/z}{1-u/z} - \frac{n'+\tilde{n}'}{2(e'\mathcal{P}')} e'^\rho \frac{1+u'z}{1-u'z} - \frac{1}{2} Q^\rho \frac{z}{1-z}, \\
A_1^{i\mu} &= \eta^{i\mu} \frac{u'}{(1-u')^2}, \quad A_2^{i\mu} = \eta^{i\mu} \frac{u'z}{(1-u'z)^2}, \\
B_1^{i\mu} &= \eta^{i\mu} \frac{u/z}{(1-u/z)^2}, \quad B_2^{i\mu} = \eta^{i\mu} \frac{u}{(1-u)^2}, \\
F^{ii'} &= \delta^{ii'} \frac{uu'}{(1-uu')^2}, \quad G^{\mu\rho} = \eta^{\mu\rho} \frac{z}{(1-z)^2}.
\end{aligned}$$

The expression for the right factor  $B_R(\bar{z})$  is completely analogous to eq.(20).

The expression for the amplitude of fig.1, eq.(6) follows from eq.(19). The factor  $C_{\mu\rho}^{ii'}$  entering eq.(7) is obtained from eq.(21) by using the relations (18).

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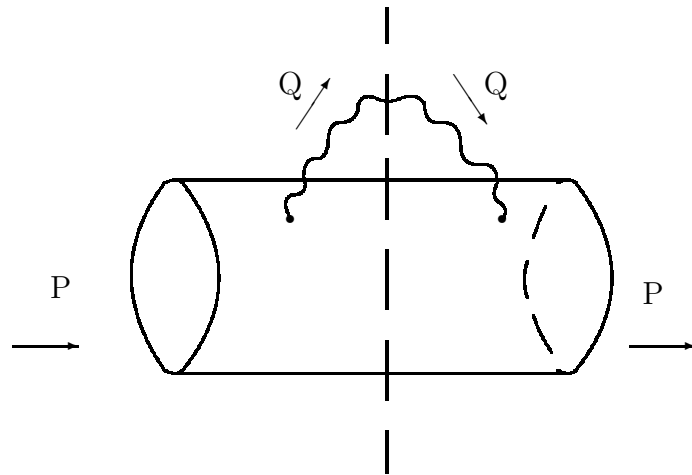


Fig. 1

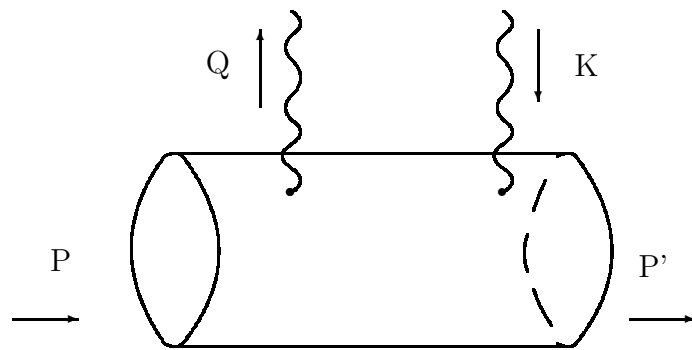


Fig. 2